

Assuming a function needs to be approximated by a polynomial of eighth degree:

$$(1) \quad F(r) \approx P(r) \equiv b_8 r^8 + b_7 r^7 + b_6 r^6 + b_5 r^5 + b_4 r^4 + b_3 r^3 + b_2 r^2 + b_1 r + b_0, \text{ where } b_i \in \mathbf{R}$$

Then a linear combination of Newton basis polynomials $n_j(r)$ can be used for finding the coefficients b_j :

$$(2) \quad P(r) = \sum_{j=0}^8 b_j r^j = \sum_{j=0}^8 a_j n_j(r)$$

For a given set of data points $(r_0, F(r_0)), \dots, (r_8, F(r_8))$, which the polynomial $P(r)$ shall interpolate, the Newton basis polynomials are formed by the following product:

$$(3) \quad n_j(r) = \prod_{i=0}^{j-1} (r - r_i) \text{ for } j > 0 \text{ and } n_0(r) = 1$$

The approximation is exact at the nodes r_i .

Therefore:

$$(4) \quad \begin{aligned} n_0(r) &= 1 \\ n_1(r) &= r - r_0 \\ n_2(r) &= (r - r_0)(r - r_1) = r^2 - r(r_0 + r_1) + r_0 r_1 \\ n_3(r) &= (r - r_0)(r - r_1)(r - r_2) = r^3 - r^2(r_0 + r_1 + r_2) + r(r_0 r_1 + r_0 r_2 + r_1 r_2) - r_0 r_1 r_2 \\ n_4(r) &= (r - r_0)(r - r_1)(r - r_2)(r - r_3) = r^4 - r^3(r_0 + r_1 + r_2 + r_3) \\ &\quad + r^2(r_0 r_1 + r_0 r_2 + r_1 r_2 + r_0 r_3 + r_1 r_3 + r_2 r_3) - r(r_0 r_1 r_2 + r_0 r_1 r_3 + r_0 r_2 r_3 + r_1 r_2 r_3) + r_0 r_1 r_2 r_3 \\ n_5(r) &= (r - r_0)(r - r_1)(r - r_2)(r - r_3)(r - r_4) = r^5 - r^4(r_0 + r_1 + r_2 + r_3 + r_4) \\ &\quad + r^3(r_0 r_1 + r_0 r_2 + r_1 r_2 + r_0 r_3 + r_1 r_3 + r_2 r_3 + r_0 r_4 + r_1 r_4 + r_2 r_4 + r_3 r_4) \\ &\quad - r^2(r_0 r_1 r_2 + r_0 r_1 r_3 + r_0 r_2 r_3 + r_1 r_2 r_3 + r_0 r_1 r_4 + r_0 r_2 r_4 + r_1 r_2 r_4 + r_0 r_3 r_4 + r_1 r_3 r_4 + r_2 r_3 r_4) \\ &\quad + r(r_0 r_1 r_2 r_3 + r_0 r_1 r_2 r_4 + r_0 r_1 r_3 r_4 + r_0 r_2 r_3 r_4 + r_1 r_2 r_3 r_4) - r_0 r_1 r_2 r_3 r_4 \\ n_6(r) &= (r - r_0)(r - r_1)(r - r_2)(r - r_3)(r - r_4)(r - r_5) = r^6 - r^5(r_0 + r_1 + r_2 + r_3 + r_4 + r_5) \\ &\quad + r^4(r_0 r_1 + r_0 r_2 + r_1 r_2 + r_0 r_3 + r_1 r_3 + r_2 r_3 + r_0 r_4 + r_1 r_4 + r_2 r_4 + r_3 r_4 + r_0 r_5 + r_1 r_5 + r_2 r_5 + r_3 r_5 + r_4 r_5) \\ &\quad - r^3(r_0 r_1 r_2 + r_0 r_1 r_3 + r_0 r_2 r_3 + r_1 r_2 r_3 + r_0 r_1 r_4 + r_0 r_2 r_4 + r_1 r_2 r_4 + r_0 r_3 r_4 + r_1 r_3 r_4 + r_2 r_3 r_4 \\ &\quad + r_0 r_1 r_5 + r_0 r_2 r_5 + r_1 r_2 r_5 + r_0 r_3 r_5 + r_1 r_3 r_5 + r_2 r_3 r_5 + r_0 r_4 r_5 + r_1 r_4 r_5 + r_2 r_4 r_5 + r_3 r_4 r_5) \\ &\quad + r^2(r_0 r_1 r_2 r_3 + r_0 r_1 r_2 r_4 + r_0 r_1 r_3 r_4 + r_0 r_2 r_3 r_4 + r_1 r_2 r_3 r_4 \\ &\quad + r_0 r_1 r_2 r_5 + r_0 r_1 r_3 r_5 + r_0 r_2 r_3 r_5 + r_1 r_2 r_3 r_5 + r_0 r_1 r_4 r_5 + r_0 r_2 r_4 r_5 + r_1 r_2 r_4 r_5 + r_0 r_3 r_4 r_5 + r_1 r_3 r_4 r_5 + r_2 r_3 r_4 r_5) \\ &\quad - r(r_0 r_1 r_2 r_3 r_4 + r_0 r_1 r_2 r_3 r_5 + r_0 r_1 r_2 r_4 r_5 + r_0 r_1 r_3 r_4 r_5 + r_0 r_2 r_3 r_4 r_5 + r_1 r_2 r_3 r_4 r_5) + r_0 r_1 r_2 r_3 r_4 r_5 \end{aligned}$$

$$\begin{aligned}
(5) \quad n_7(r) &= (r-r_0)(r-r_1)(r-r_2)(r-r_3)(r-r_4)(r-r_5)(r-r_6) \\
&= r^7 - r^6(r_0+r_1+r_2+r_3+r_4+r_5+r_6) \\
&\quad + r^5(r_0r_1+r_0r_2+r_1r_2+r_0r_3+r_1r_3+r_2r_3+r_0r_4+r_1r_4+r_2r_4+r_3r_4 \\
&\quad + r_0r_5+r_1r_5+r_2r_5+r_3r_5+r_4r_5+r_0r_6+r_1r_6+r_2r_6+r_3r_6+r_4r_6+r_5r_6) \\
&\quad - r^4(r_0r_1r_2+r_0r_1r_3+r_0r_2r_3+r_1r_2r_3+r_0r_1r_4+r_0r_2r_4+r_1r_2r_4+r_0r_3r_4+r_1r_3r_4+r_2r_3r_4 \\
&\quad + r_0r_1r_5+r_0r_2r_5+r_1r_2r_5+r_0r_3r_5+r_1r_3r_5+r_2r_3r_5+r_0r_4r_5+r_1r_4r_5+r_2r_4r_5+r_3r_4r_5 \\
&\quad + r_0r_1r_6+r_0r_2r_6+r_1r_2r_6+r_0r_3r_6+r_1r_3r_6+r_2r_3r_6+r_0r_4r_6+r_1r_4r_6+r_2r_4r_6+r_3r_4r_6 \\
&\quad + r_0r_5r_6+r_1r_5r_6+r_2r_5r_6+r_3r_5r_6+r_4r_5r_6) \\
&\quad + r^3(r_0r_1r_2r_3+r_0r_1r_2r_4+r_0r_1r_3r_4+r_0r_2r_3r_4+r_1r_2r_3r_4 \\
&\quad + r_0r_1r_2r_5+r_0r_1r_3r_5+r_0r_2r_3r_5+r_1r_2r_3r_5+r_0r_1r_4r_5+r_0r_2r_4r_5+r_1r_2r_4r_5+r_0r_3r_4r_5+r_1r_3r_4r_5+r_2r_3r_4r_5 \\
&\quad + r_0r_1r_2r_6+r_0r_1r_3r_6+r_0r_2r_3r_6+r_1r_2r_3r_6+r_0r_1r_4r_6+r_0r_2r_4r_6+r_1r_2r_4r_6+r_0r_3r_4r_6+r_1r_3r_4r_6+r_2r_3r_4r_6 \\
&\quad + r_0r_1r_5r_6+r_0r_2r_5r_6+r_1r_2r_5r_6+r_0r_3r_5r_6+r_1r_3r_5r_6+r_2r_3r_5r_6+r_0r_4r_5r_6+r_1r_4r_5r_6+r_2r_4r_5r_6+r_3r_4r_5r_6) \\
&\quad - r^2(r_0r_1r_2r_3r_4+r_0r_1r_2r_3r_5+r_0r_1r_2r_4r_5+r_0r_1r_3r_4r_5+r_0r_2r_3r_4r_5+r_1r_2r_3r_4r_5 \\
&\quad + r_0r_1r_2r_3r_6+r_0r_1r_2r_4r_6+r_0r_1r_3r_4r_6+r_0r_2r_3r_4r_6+r_1r_2r_3r_4r_6 \\
&\quad + r_0r_1r_2r_5r_6+r_0r_1r_3r_5r_6+r_0r_2r_3r_5r_6+r_1r_2r_3r_5r_6+r_0r_1r_4r_5r_6 \\
&\quad + r_0r_2r_4r_5r_6+r_1r_2r_4r_5r_6+r_0r_3r_4r_5r_6+r_1r_3r_4r_5r_6+r_2r_3r_4r_5r_6) \\
&\quad + r(r_0r_1r_2r_3r_4r_5+r_0r_1r_2r_3r_4r_6+r_0r_1r_2r_3r_5r_6+r_0r_1r_2r_4r_5r_6+r_0r_1r_3r_4r_5r_6+r_0r_2r_3r_4r_5r_6+r_1r_2r_3r_4r_5r_6) \\
&\quad - r_0r_1r_2r_3r_4r_5r_6
\end{aligned}$$

$$\begin{aligned}
(6) \quad n_8(r) &= (r-r_0)(r-r_1)(r-r_2)(r-r_3)(r-r_4)(r-r_5)(r-r_6)(r-r_7) \\
&= r^8 - r^7(r_0+r_1+r_2+r_3+r_4+r_5+r_6+r_7) \\
&\quad + r^6(r_0r_1+r_0r_2+r_1r_2+r_0r_3+r_1r_3+r_2r_3+r_0r_4+r_1r_4+r_2r_4+r_3r_4+r_0r_5+r_1r_5+r_2r_5+r_3r_5+r_4r_5 \\
&\quad + r_0r_6+r_1r_6+r_2r_6+r_3r_6+r_4r_6+r_5r_6+r_0r_7+r_1r_7+r_2r_7+r_3r_7+r_4r_7+r_5r_7+r_6r_7) \\
&\quad - r^5(r_0r_1r_2+r_0r_1r_3+r_0r_2r_3+r_1r_2r_3+r_0r_1r_4+r_0r_2r_4+r_1r_2r_4+r_0r_3r_4+r_1r_3r_4+r_2r_3r_4 \\
&\quad + r_0r_1r_5+r_0r_2r_5+r_1r_2r_5+r_0r_3r_5+r_1r_3r_5+r_2r_3r_5+r_0r_4r_5+r_1r_4r_5+r_2r_4r_5+r_3r_4r_5 \\
&\quad + r_0r_1r_6+r_0r_2r_6+r_1r_2r_6+r_0r_3r_6+r_1r_3r_6+r_2r_3r_6+r_0r_4r_6+r_1r_4r_6+r_2r_4r_6+r_3r_4r_6 \\
&\quad + r_0r_5r_6+r_1r_5r_6+r_2r_5r_6+r_3r_5r_6+r_4r_5r_6 \\
&\quad + r_0r_1r_7+r_0r_2r_7+r_1r_2r_7+r_0r_3r_7+r_1r_3r_7+r_2r_3r_7+r_0r_4r_7+r_1r_4r_7+r_2r_4r_7+r_3r_4r_7 \\
&\quad + r_0r_5r_7+r_1r_5r_7+r_2r_5r_7+r_3r_5r_7+r_4r_5r_7+r_0r_6r_7+r_1r_6r_7+r_2r_6r_7+r_3r_6r_7+r_4r_6r_7+r_5r_6r_7) \\
&\quad + r^4(r_0r_1r_2r_3+r_0r_1r_2r_4+r_0r_1r_3r_4+r_0r_2r_3r_4+r_1r_2r_3r_4 \\
&\quad + r_0r_1r_2r_5+r_0r_1r_3r_5+r_0r_2r_3r_5+r_1r_2r_3r_5+r_0r_1r_4r_5+r_0r_2r_4r_5+r_1r_2r_4r_5+r_0r_3r_4r_5+r_1r_3r_4r_5+r_2r_3r_4r_5 \\
&\quad + r_0r_1r_2r_6+r_0r_1r_3r_6+r_0r_2r_3r_6+r_1r_2r_3r_6+r_0r_1r_4r_6+r_0r_2r_4r_6+r_1r_2r_4r_6+r_0r_3r_4r_6+r_1r_3r_4r_6+r_2r_3r_4r_6 \\
&\quad + r_0r_1r_5r_6+r_0r_2r_5r_6+r_1r_2r_5r_6+r_0r_3r_5r_6+r_1r_3r_5r_6+r_2r_3r_5r_6+r_0r_4r_5r_6+r_1r_4r_5r_6+r_2r_4r_5r_6+r_3r_4r_5r_6 \\
&\quad + r_0r_1r_2r_7+r_0r_1r_3r_7+r_0r_2r_3r_7+r_1r_2r_3r_7+r_0r_1r_4r_7+r_0r_2r_4r_7+r_1r_2r_4r_7+r_0r_3r_4r_7+r_1r_3r_4r_7+r_2r_3r_4r_7 \\
&\quad + r_0r_1r_5r_7+r_0r_2r_5r_7+r_1r_2r_5r_7+r_0r_3r_5r_7+r_1r_3r_5r_7+r_2r_3r_5r_7+r_0r_4r_5r_7+r_1r_4r_5r_7+r_2r_4r_5r_7+r_3r_4r_5r_7 \\
&\quad + r_0r_1r_6r_7+r_0r_2r_6r_7+r_1r_2r_6r_7+r_0r_3r_6r_7+r_1r_3r_6r_7+r_2r_3r_6r_7+r_0r_4r_6r_7+r_1r_4r_6r_7+r_2r_4r_6r_7+r_3r_4r_6r_7 \\
&\quad + r_0r_5r_6r_7+r_1r_5r_6r_7+r_2r_5r_6r_7+r_3r_5r_6r_7+r_4r_5r_6r_7) \\
&\quad - r^3(r_0r_1r_2r_3r_4+r_0r_1r_2r_3r_5+r_0r_1r_2r_4r_5+r_0r_1r_3r_4r_5+r_0r_2r_3r_4r_5+r_1r_2r_3r_4r_5 \\
&\quad + r_0r_1r_2r_3r_6+r_0r_1r_2r_4r_6+r_0r_1r_3r_4r_6+r_0r_2r_3r_4r_6+r_1r_2r_3r_4r_6 \\
&\quad + r_0r_1r_2r_5r_6+r_0r_1r_3r_5r_6+r_0r_2r_3r_5r_6+r_1r_2r_3r_5r_6+r_0r_1r_4r_5r_6 \\
&\quad + r_0r_2r_4r_5r_6+r_1r_2r_4r_5r_6+r_0r_3r_4r_5r_6+r_1r_3r_4r_5r_6+r_2r_3r_4r_5r_6 \\
&\quad + r_0r_1r_2r_3r_7+r_0r_1r_2r_4r_7+r_0r_1r_3r_4r_7+r_0r_2r_3r_4r_7+r_1r_2r_3r_4r_7 \\
&\quad + r_0r_1r_2r_5r_7+r_0r_1r_3r_5r_7+r_0r_2r_3r_5r_7+r_1r_2r_3r_5r_7+r_0r_1r_4r_5r_7 \\
&\quad + r_0r_2r_4r_5r_7+r_1r_2r_4r_5r_7+r_0r_3r_4r_5r_7+r_1r_3r_4r_5r_7+r_2r_3r_4r_5r_7 \\
&\quad + r_0r_1r_2r_6r_7+r_0r_1r_3r_6r_7+r_0r_2r_3r_6r_7+r_1r_2r_3r_6r_7 \\
&\quad + r_0r_3r_4r_6r_7+r_1r_3r_4r_6r_7+r_2r_3r_4r_6r_7+r_0r_1r_5r_6r_7+r_0r_2r_5r_6r_7+r_1r_2r_5r_6r_7 \\
&\quad + r_0r_3r_5r_6r_7+r_1r_3r_5r_6r_7+r_2r_3r_5r_6r_7+r_0r_4r_5r_6r_7+r_1r_4r_5r_6r_7+r_2r_4r_5r_6r_7+r_3r_4r_5r_6r_7) \\
&\quad + r^2(r_0r_1r_2r_3r_4r_5+r_0r_1r_2r_3r_4r_6+r_0r_1r_2r_3r_5r_6+r_0r_1r_2r_4r_5r_6+r_0r_1r_3r_4r_5r_6+r_0r_2r_3r_4r_5r_6+r_1r_2r_3r_4r_5r_6 \\
&\quad + r_0r_1r_2r_3r_4r_7+r_0r_1r_2r_3r_5r_7+r_0r_1r_2r_4r_5r_7+r_0r_1r_3r_4r_5r_7+r_0r_2r_3r_4r_5r_7+r_1r_2r_3r_4r_5r_7+r_0r_1r_2r_3r_6r_7 \\
&\quad + r_0r_1r_2r_4r_6r_7+r_0r_1r_3r_4r_6r_7+r_0r_2r_3r_4r_6r_7+r_1r_2r_3r_4r_6r_7+r_0r_1r_2r_5r_6r_7+r_0r_1r_3r_5r_6r_7+r_0r_2r_3r_5r_6r_7+r_1r_2r_3r_5r_6r_7 \\
&\quad + r_0r_1r_4r_5r_6r_7+r_0r_2r_4r_5r_6r_7+r_1r_2r_4r_5r_6r_7+r_0r_3r_4r_5r_6r_7+r_1r_3r_4r_5r_6r_7+r_2r_3r_4r_5r_6r_7) \\
&\quad - r(r_0r_1r_2r_3r_4r_5+r_0r_1r_2r_3r_4r_6+r_0r_1r_2r_3r_5r_6+r_0r_1r_2r_4r_5r_6+r_0r_1r_3r_4r_5r_6+r_0r_2r_3r_4r_5r_6+r_1r_2r_3r_4r_5r_6 \\
&\quad + r_0r_1r_2r_3r_4r_7+r_0r_1r_2r_3r_5r_7+r_0r_1r_2r_4r_5r_7+r_0r_1r_3r_4r_5r_7+r_0r_2r_3r_4r_5r_7+r_1r_2r_3r_4r_5r_7 \\
&\quad + r_0r_1r_2r_4r_6r_7+r_0r_1r_3r_4r_6r_7+r_0r_2r_3r_4r_6r_7+r_1r_2r_3r_4r_6r_7+r_0r_1r_2r_5r_6r_7+r_0r_1r_3r_5r_6r_7+r_0r_2r_3r_5r_6r_7+r_1r_2r_3r_5r_6r_7 \\
&\quad + r_0r_1r_4r_5r_6r_7+r_0r_2r_4r_5r_6r_7+r_1r_2r_4r_5r_6r_7+r_0r_3r_4r_5r_6r_7+r_1r_3r_4r_5r_6r_7+r_2r_3r_4r_5r_6r_7) \\
&\quad + r_0r_1r_2r_3r_4r_5r_6r_7
\end{aligned}$$

Putting (2), (4), (5) and (6) together leads to the desired coefficients b_j :

$$\begin{aligned}
 (7) \quad b_0 &= a_0 - a_1 r_0 + a_2 r_0 r_1 - a_3 r_0 r_1 r_2 + a_4 r_0 r_1 r_2 r_3 - a_5 r_0 r_1 r_2 r_3 r_4 + a_6 r_0 r_1 r_2 r_3 r_4 r_5 - a_7 r_0 r_1 r_2 r_3 r_4 r_5 r_6 \\
 &\quad + a_8 r_0 r_1 r_2 r_3 r_4 r_5 r_6 r_7 \\
 b_1 &= a_1 - a_2 (r_0 + r_1) + a_3 (r_0 r_1 + r_0 r_2 + r_1 r_2) - a_4 (r_0 r_1 r_2 + r_0 r_1 r_3 + r_0 r_2 r_3 + r_1 r_2 r_3) \\
 &\quad + a_5 (r_0 r_1 r_2 r_3 + r_0 r_1 r_2 r_4 + r_0 r_1 r_3 r_4 + r_0 r_2 r_3 r_4 + r_1 r_2 r_3 r_4) \\
 &\quad - a_6 (r_0 r_1 r_2 r_3 r_4 + r_0 r_1 r_2 r_3 r_5 + r_0 r_1 r_2 r_4 r_5 + r_0 r_1 r_3 r_4 r_5 + r_0 r_2 r_3 r_4 r_5 + r_1 r_2 r_3 r_4 r_5) \\
 &\quad + a_7 (r_0 r_1 r_2 r_3 r_4 r_5 + r_0 r_1 r_2 r_3 r_4 r_6 + r_0 r_1 r_2 r_3 r_5 r_6 + r_0 r_1 r_2 r_4 r_5 r_6 + r_0 r_1 r_3 r_4 r_5 r_6 + r_0 r_2 r_3 r_4 r_5 r_6 + r_1 r_2 r_3 r_4 r_5 r_6) \\
 &\quad - a_8 (r_0 r_1 r_2 r_3 r_4 r_5 r_6 + r_0 r_1 r_2 r_3 r_4 r_5 r_7 + r_0 r_1 r_2 r_3 r_4 r_6 r_7 \\
 &\quad + r_0 r_1 r_2 r_3 r_5 r_6 r_7 + r_0 r_1 r_2 r_4 r_5 r_6 r_7 + r_0 r_1 r_3 r_4 r_5 r_6 r_7 + r_0 r_2 r_3 r_4 r_5 r_6 r_7 + r_1 r_2 r_3 r_4 r_5 r_6 r_7) \\
 b_2 &= a_2 - a_3 (r_0 + r_1 + r_2) + a_4 (r_0 r_1 + r_0 r_2 + r_1 r_2 + r_0 r_3 + r_1 r_3 + r_2 r_3) \\
 &\quad - a_5 (r_0 r_1 r_2 + r_0 r_1 r_3 + r_0 r_2 r_3 + r_1 r_2 r_3 + r_0 r_1 r_4 + r_0 r_2 r_4 + r_1 r_2 r_4 + r_0 r_3 r_4 + r_1 r_3 r_4 + r_2 r_3 r_4) \\
 &\quad + a_6 (r_0 r_1 r_2 r_3 + r_0 r_1 r_2 r_4 + r_0 r_1 r_3 r_4 + r_0 r_2 r_3 r_4 + r_1 r_2 r_3 r_4 \\
 &\quad + r_0 r_1 r_2 r_5 + r_0 r_1 r_3 r_5 + r_0 r_2 r_3 r_5 + r_1 r_2 r_3 r_5 + r_0 r_1 r_4 r_5 + r_0 r_2 r_4 r_5 + r_1 r_2 r_4 r_5 + r_0 r_3 r_4 r_5 + r_1 r_3 r_4 r_5 + r_2 r_3 r_4 r_5) \\
 &\quad - a_7 (r_0 r_1 r_2 r_3 r_4 + r_0 r_1 r_2 r_3 r_5 + r_0 r_1 r_2 r_4 r_5 + r_0 r_1 r_3 r_4 r_5 + r_0 r_2 r_3 r_4 r_5 + r_1 r_2 r_3 r_4 r_5 \\
 &\quad + r_0 r_1 r_2 r_3 r_6 + r_0 r_1 r_2 r_4 r_6 + r_0 r_1 r_3 r_4 r_6 + r_0 r_2 r_3 r_4 r_6 + r_1 r_2 r_3 r_4 r_6 \\
 &\quad + r_0 r_1 r_2 r_5 r_6 + r_0 r_1 r_3 r_5 r_6 + r_0 r_2 r_3 r_5 r_6 + r_1 r_2 r_3 r_5 r_6 + r_0 r_1 r_4 r_5 r_6 \\
 &\quad + r_0 r_2 r_4 r_5 r_6 + r_1 r_2 r_4 r_5 r_6 + r_0 r_3 r_4 r_5 r_6 + r_1 r_3 r_4 r_5 r_6 + r_2 r_3 r_4 r_5 r_6) \\
 &\quad + a_8 (r_0 r_1 r_2 r_3 r_4 r_5 + r_0 r_1 r_2 r_3 r_4 r_6 + r_0 r_1 r_2 r_3 r_5 r_6 + r_0 r_1 r_2 r_4 r_5 r_6 + r_0 r_1 r_3 r_4 r_5 r_6 + r_0 r_2 r_3 r_4 r_5 r_6 + r_1 r_2 r_3 r_4 r_5 r_6 \\
 &\quad + r_0 r_1 r_2 r_3 r_4 r_7 + r_0 r_1 r_2 r_3 r_5 r_7 + r_0 r_1 r_2 r_4 r_5 r_7 + r_0 r_1 r_3 r_4 r_5 r_7 + r_0 r_2 r_3 r_4 r_5 r_7 + r_1 r_2 r_3 r_4 r_5 r_7 \\
 &\quad + r_0 r_1 r_2 r_3 r_6 r_7 + r_0 r_1 r_2 r_4 r_6 r_7 + r_0 r_1 r_3 r_4 r_6 r_7 + r_0 r_2 r_3 r_4 r_6 r_7 + r_1 r_2 r_3 r_4 r_6 r_7 \\
 &\quad + r_0 r_1 r_2 r_5 r_6 r_7 + r_0 r_1 r_3 r_5 r_6 r_7 + r_0 r_2 r_3 r_5 r_6 r_7 + r_1 r_2 r_3 r_5 r_6 r_7 + r_0 r_1 r_4 r_5 r_6 r_7 \\
 &\quad + r_0 r_2 r_4 r_5 r_6 r_7 + r_1 r_2 r_4 r_5 r_6 r_7 + r_0 r_3 r_4 r_5 r_6 r_7 + r_1 r_3 r_4 r_5 r_6 r_7 + r_2 r_3 r_4 r_5 r_6 r_7) \\
 b_3 &= a_3 - a_4 (r_0 + r_1 + r_2 + r_3) + a_5 (r_0 r_1 + r_0 r_2 + r_1 r_2 + r_0 r_3 + r_1 r_3 + r_2 r_3 + r_0 r_4 + r_1 r_4 + r_2 r_4 + r_3 r_4) \\
 &\quad - a_6 (r_0 r_1 r_2 + r_0 r_1 r_3 + r_0 r_2 r_3 + r_1 r_2 r_3 + r_0 r_1 r_4 + r_0 r_2 r_4 + r_1 r_2 r_4 + r_0 r_3 r_4 + r_1 r_3 r_4 + r_2 r_3 r_4 \\
 &\quad + r_0 r_1 r_5 + r_0 r_2 r_5 + r_1 r_2 r_5 + r_0 r_3 r_5 + r_1 r_3 r_5 + r_2 r_3 r_5 + r_0 r_4 r_5 + r_1 r_4 r_5 + r_2 r_4 r_5 + r_3 r_4 r_5) \\
 &\quad + a_7 (r_0 r_1 r_2 r_3 + r_0 r_1 r_2 r_4 + r_0 r_1 r_3 r_4 + r_0 r_2 r_3 r_4 + r_1 r_2 r_3 r_4 \\
 &\quad + r_0 r_1 r_2 r_5 + r_0 r_1 r_3 r_5 + r_0 r_2 r_3 r_5 + r_1 r_2 r_3 r_5 + r_0 r_1 r_4 r_5 + r_0 r_2 r_4 r_5 + r_1 r_2 r_4 r_5 + r_0 r_3 r_4 r_5 + r_1 r_3 r_4 r_5 + r_2 r_3 r_4 r_5 \\
 &\quad + r_0 r_1 r_2 r_6 + r_0 r_1 r_3 r_6 + r_0 r_2 r_3 r_6 + r_1 r_2 r_3 r_6 + r_0 r_1 r_4 r_6 + r_0 r_2 r_4 r_6 + r_1 r_2 r_4 r_6 + r_0 r_3 r_4 r_6 + r_1 r_3 r_4 r_6 + r_2 r_3 r_4 r_6 \\
 &\quad + r_0 r_1 r_5 r_6 + r_0 r_2 r_5 r_6 + r_1 r_2 r_5 r_6 + r_0 r_3 r_5 r_6 + r_1 r_3 r_5 r_6 + r_2 r_3 r_5 r_6 + r_0 r_4 r_5 r_6 + r_1 r_4 r_5 r_6 + r_2 r_4 r_5 r_6 + r_3 r_4 r_5 r_6) \\
 &\quad - a_8 (r_0 r_1 r_2 r_3 r_4 + r_0 r_1 r_2 r_3 r_5 + r_0 r_1 r_2 r_4 r_5 + r_0 r_1 r_3 r_4 r_5 + r_0 r_2 r_3 r_4 r_5 + r_1 r_2 r_3 r_4 r_5 \\
 &\quad + r_0 r_1 r_2 r_3 r_6 + r_0 r_1 r_2 r_4 r_6 + r_0 r_1 r_3 r_4 r_6 + r_0 r_2 r_3 r_4 r_6 + r_1 r_2 r_3 r_4 r_6 + r_0 r_1 r_2 r_5 r_6 + r_0 r_1 r_3 r_5 r_6 + r_0 r_2 r_3 r_5 r_6 \\
 &\quad + r_1 r_2 r_3 r_5 r_6 + r_0 r_1 r_4 r_5 r_6 + r_0 r_2 r_4 r_5 r_6 + r_1 r_2 r_4 r_5 r_6 + r_0 r_3 r_4 r_5 r_6 + r_1 r_3 r_4 r_5 r_6 + r_2 r_3 r_4 r_5 r_6 \\
 &\quad + r_0 r_1 r_2 r_3 r_7 + r_0 r_1 r_2 r_4 r_7 + r_0 r_1 r_3 r_4 r_7 + r_0 r_2 r_3 r_4 r_7 + r_1 r_2 r_3 r_4 r_7 + r_0 r_1 r_2 r_5 r_7 + r_0 r_1 r_3 r_5 r_7 + r_0 r_2 r_3 r_5 r_7 \\
 &\quad + r_1 r_2 r_3 r_5 r_7 + r_0 r_1 r_4 r_5 r_7 + r_0 r_2 r_4 r_5 r_7 + r_1 r_2 r_4 r_5 r_7 + r_0 r_3 r_4 r_5 r_7 + r_1 r_3 r_4 r_5 r_7 + r_2 r_3 r_4 r_5 r_7 + r_0 r_1 r_2 r_6 r_7 + r_0 r_1 r_3 r_6 r_7 \\
 &\quad + r_0 r_2 r_3 r_6 r_7 + r_1 r_2 r_3 r_6 r_7 + r_0 r_1 r_4 r_6 r_7 + r_0 r_2 r_4 r_6 r_7 + r_1 r_2 r_4 r_6 r_7 + r_0 r_3 r_4 r_6 r_7 + r_1 r_3 r_4 r_6 r_7 + r_2 r_3 r_4 r_6 r_7 + r_0 r_1 r_5 r_6 r_7 \\
 &\quad + r_0 r_2 r_5 r_6 r_7 + r_1 r_2 r_5 r_6 r_7 + r_0 r_3 r_5 r_6 r_7 + r_1 r_3 r_5 r_6 r_7 + r_2 r_3 r_5 r_6 r_7 + r_0 r_4 r_5 r_6 r_7 + r_1 r_4 r_5 r_6 r_7 + r_2 r_4 r_5 r_6 r_7 + r_3 r_4 r_5 r_6 r_7)
 \end{aligned}$$

$$\begin{aligned}
(8) \quad & b_4 = a_4 - a_5(r_0 + r_1 + r_2 + r_3 + r_4) \\
& + a_6(r_0r_1 + r_0r_2 + r_1r_2 + r_0r_3 + r_1r_3 + r_2r_3 + r_0r_4 + r_1r_4 + r_2r_4 + r_3r_4 + r_0r_5 + r_1r_5 + r_2r_5 + r_3r_5 + r_4r_5) \\
& - a_7(r_0r_1r_2 + r_0r_1r_3 + r_0r_2r_3 + r_1r_2r_3 + r_0r_1r_4 + r_0r_2r_4 + r_1r_2r_4 + r_0r_3r_4 + r_1r_3r_4 + r_2r_3r_4 \\
& + r_0r_1r_5 + r_0r_2r_5 + r_1r_2r_5 + r_0r_3r_5 + r_1r_3r_5 + r_2r_3r_5 + r_0r_4r_5 + r_1r_4r_5 + r_2r_4r_5 + r_3r_4r_5 \\
& + r_0r_1r_6 + r_0r_2r_6 + r_1r_2r_6 + r_0r_3r_6 + r_1r_3r_6 + r_2r_3r_6 + r_0r_4r_6 + r_1r_4r_6 + r_2r_4r_6 + r_3r_4r_6 \\
& + r_0r_5r_6 + r_1r_5r_6 + r_2r_5r_6 + r_3r_5r_6 + r_4r_5r_6) \\
& + a_8(r_0r_1r_2r_3 + r_0r_1r_2r_4 + r_0r_1r_3r_4 + r_0r_2r_3r_4 + r_1r_2r_3r_4 \\
& + r_0r_1r_2r_5 + r_0r_1r_3r_5 + r_0r_2r_3r_5 + r_1r_2r_3r_5 + r_0r_1r_4r_5 + r_0r_2r_4r_5 + r_1r_2r_4r_5 + r_0r_3r_4r_5 + r_1r_3r_4r_5 + r_2r_3r_4r_5 \\
& + r_0r_1r_2r_6 + r_0r_1r_3r_6 + r_0r_2r_3r_6 + r_1r_2r_3r_6 + r_0r_1r_4r_6 + r_0r_2r_4r_6 + r_1r_2r_4r_6 + r_0r_3r_4r_6 + r_1r_3r_4r_6 + r_2r_3r_4r_6 \\
& + r_0r_1r_5r_6 + r_0r_2r_5r_6 + r_1r_2r_5r_6 + r_0r_3r_5r_6 + r_1r_3r_5r_6 + r_2r_3r_5r_6 + r_0r_4r_5r_6 + r_1r_4r_5r_6 + r_2r_4r_5r_6 + r_3r_4r_5r_6 \\
& + r_0r_1r_2r_7 + r_0r_1r_3r_7 + r_0r_2r_3r_7 + r_1r_2r_3r_7 + r_0r_1r_4r_7 + r_0r_2r_4r_7 + r_1r_2r_4r_7 + r_0r_3r_4r_7 + r_1r_3r_4r_7 + r_2r_3r_4r_7 \\
& + r_0r_1r_5r_7 + r_0r_2r_5r_7 + r_1r_2r_5r_7 + r_0r_3r_5r_7 + r_1r_3r_5r_7 + r_2r_3r_5r_7 + r_0r_4r_5r_7 + r_1r_4r_5r_7 + r_2r_4r_5r_7 + r_3r_4r_5r_7 \\
& + r_0r_1r_6r_7 + r_0r_2r_6r_7 + r_1r_2r_6r_7 + r_0r_3r_6r_7 + r_1r_3r_6r_7 + r_2r_3r_6r_7 + r_0r_4r_6r_7 + r_1r_4r_6r_7 + r_2r_4r_6r_7 + r_3r_4r_6r_7 \\
& + r_0r_5r_6r_7 + r_1r_5r_6r_7 + r_2r_5r_6r_7 + r_3r_5r_6r_7 + r_4r_5r_6r_7) \\
& b_5 = a_5 - a_6(r_0 + r_1 + r_2 + r_3 + r_4 + r_5) \\
& + a_7(r_0r_1 + r_0r_2 + r_1r_2 + r_0r_3 + r_1r_3 + r_2r_3 + r_0r_4 + r_1r_4 + r_2r_4 + r_3r_4 \\
& + r_0r_5 + r_1r_5 + r_2r_5 + r_3r_5 + r_4r_5 + r_0r_6 + r_1r_6 + r_2r_6 + r_3r_6 + r_4r_6 + r_5r_6) \\
& - a_8(r_0r_1r_2 + r_0r_1r_3 + r_0r_2r_3 + r_1r_2r_3 + r_0r_1r_4 + r_0r_2r_4 + r_1r_2r_4 + r_0r_3r_4 + r_1r_3r_4 + r_2r_3r_4 + r_0r_1r_5 \\
& + r_0r_2r_5 + r_1r_2r_5 + r_0r_3r_5 + r_1r_3r_5 + r_2r_3r_5 + r_0r_4r_5 + r_1r_4r_5 + r_2r_4r_5 + r_3r_4r_5 + r_0r_1r_6 + r_0r_2r_6 + r_1r_2r_6 \\
& + r_0r_3r_6 + r_1r_3r_6 + r_2r_3r_6 + r_0r_4r_6 + r_1r_4r_6 + r_2r_4r_6 + r_3r_4r_6 + r_0r_5r_6 + r_1r_5r_6 + r_2r_5r_6 + r_3r_5r_6 + r_4r_5r_6 \\
& + r_0r_1r_7 + r_0r_2r_7 + r_1r_2r_7 + r_0r_3r_7 + r_1r_3r_7 + r_2r_3r_7 + r_0r_4r_7 + r_1r_4r_7 + r_2r_4r_7 + r_3r_4r_7 \\
& + r_0r_5r_7 + r_1r_5r_7 + r_2r_5r_7 + r_3r_5r_7 + r_4r_5r_7 + r_0r_6r_7 + r_1r_6r_7 + r_2r_6r_7 + r_3r_6r_7 + r_4r_6r_7 + r_5r_6r_7) \\
& b_6 = a_6 - a_7(r_0 + r_1 + r_2 + r_3 + r_4 + r_5 + r_6) \\
& + a_8(r_0r_1 + r_0r_2 + r_1r_2 + r_0r_3 + r_1r_3 + r_2r_3 + r_0r_4 + r_1r_4 + r_2r_4 + r_3r_4 + r_0r_5 + r_1r_5 + r_2r_5 + r_3r_5 + r_4r_5 \\
& + r_0r_6 + r_1r_6 + r_2r_6 + r_3r_6 + r_4r_6 + r_5r_6 + r_0r_7 + r_1r_7 + r_2r_7 + r_3r_7 + r_4r_7 + r_5r_7 + r_6r_7) \\
& b_7 = a_7 - a_8(r_0 + r_1 + r_2 + r_3 + r_4 + r_5 + r_6 + r_7) \\
& b_8 = a_8
\end{aligned}$$

The coefficients a_j are the divided differences:

$$(9) \quad a_j = [F(r_0), F(r_1), \dots, F(r_j)]$$

In particular:

$$(10) \quad a_0 = F(r_0)$$

$$a_1 = \frac{F(r_1) - F(r_0)}{r_1 - r_0}$$

$$a_2 = \frac{F(r_2) - F(r_1)}{(r_2 - r_1)(r_2 - r_0)} - \frac{F(r_1) - F(r_0)}{(r_1 - r_0)(r_2 - r_0)}$$

$$a_3 = \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_3 - r_1)(r_3 - r_0)} - \frac{F(r_2) - F(r_1)}{(r_2 - r_1)(r_3 - r_1)(r_3 - r_0)} - \frac{F(r_2) - F(r_1)}{(r_2 - r_1)(r_2 - r_0)(r_3 - r_0)} + \frac{F(r_1) - F(r_0)}{(r_1 - r_0)(r_2 - r_0)(r_3 - r_0)}$$

$$a_4 = \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_4 - r_1)(r_4 - r_0)} - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_4 - r_1)(r_4 - r_0)} - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_3 - r_1)(r_4 - r_1)(r_4 - r_0)} + \frac{F(r_2) - F(r_1)}{(r_2 - r_1)(r_3 - r_1)(r_4 - r_1)(r_4 - r_0)} - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_3 - r_1)(r_3 - r_0)(r_4 - r_0)} + \frac{F(r_2) - F(r_1)}{(r_2 - r_1)(r_3 - r_1)(r_3 - r_0)(r_4 - r_0)} + \frac{F(r_2) - F(r_1)}{(r_2 - r_1)(r_2 - r_0)(r_3 - r_0)(r_4 - r_0)} - \frac{F(r_1) - F(r_0)}{(r_1 - r_0)(r_2 - r_0)(r_3 - r_0)(r_4 - r_0)}$$

$$(11) \quad a_5 = \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_5 - r_2)(r_5 - r_1)(r_5 - r_0)} - \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_5 - r_2)(r_5 - r_1)(r_5 - r_0)} - \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_5 - r_2)(r_5 - r_1)(r_5 - r_0)} + \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_5 - r_2)(r_5 - r_1)(r_5 - r_0)} - \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_4 - r_1)(r_5 - r_1)(r_5 - r_0)} + \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_4 - r_1)(r_5 - r_1)(r_5 - r_0)} + \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_3 - r_1)(r_4 - r_1)(r_5 - r_1)(r_5 - r_0)} - \frac{F(r_2) - F(r_1)}{(r_2 - r_1)(r_3 - r_1)(r_4 - r_1)(r_5 - r_1)(r_5 - r_0)} - \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_4 - r_1)(r_4 - r_0)(r_5 - r_0)} + \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_4 - r_1)(r_4 - r_0)(r_5 - r_0)} + \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_3 - r_1)(r_4 - r_1)(r_4 - r_0)(r_5 - r_0)} - \frac{F(r_2) - F(r_1)}{(r_2 - r_1)(r_3 - r_1)(r_4 - r_1)(r_4 - r_0)(r_5 - r_0)} + \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_3 - r_1)(r_3 - r_0)(r_4 - r_0)(r_5 - r_0)} - \frac{F(r_2) - F(r_1)}{(r_2 - r_1)(r_3 - r_1)(r_3 - r_0)(r_4 - r_0)(r_5 - r_0)} - \frac{F(r_2) - F(r_1)}{(r_2 - r_1)(r_2 - r_0)(r_3 - r_0)(r_4 - r_0)(r_5 - r_0)} + \frac{F(r_1) - F(r_0)}{(r_1 - r_0)(r_2 - r_0)(r_3 - r_0)(r_4 - r_0)(r_5 - r_0)}$$

(12) a_6

$$\begin{aligned} &= \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_6 - r_4)(r_6 - r_3)(r_6 - r_2)(r_6 - r_1)(r_6 - r_0)} - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_6 - r_4)(r_6 - r_3)(r_6 - r_2)(r_6 - r_1)(r_6 - r_0)} \\ &- \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_6 - r_3)(r_6 - r_2)(r_6 - r_1)(r_6 - r_0)} + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_6 - r_3)(r_6 - r_2)(r_6 - r_1)(r_6 - r_0)} \\ &- \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_5 - r_2)(r_6 - r_2)(r_6 - r_1)(r_6 - r_0)} + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_5 - r_2)(r_6 - r_2)(r_6 - r_1)(r_6 - r_0)} \\ &+ \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_5 - r_2)(r_6 - r_2)(r_6 - r_1)(r_6 - r_0)} - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_5 - r_2)(r_6 - r_2)(r_6 - r_1)(r_6 - r_0)} \\ &- \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_5 - r_2)(r_5 - r_1)(r_6 - r_1)(r_6 - r_0)} + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_5 - r_2)(r_5 - r_1)(r_6 - r_1)(r_6 - r_0)} \\ &+ \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_5 - r_2)(r_5 - r_1)(r_6 - r_1)(r_6 - r_0)} - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_5 - r_2)(r_5 - r_1)(r_6 - r_1)(r_6 - r_0)} \\ &+ \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_4 - r_1)(r_5 - r_1)(r_6 - r_1)(r_6 - r_0)} - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_4 - r_1)(r_5 - r_1)(r_6 - r_1)(r_6 - r_0)} \\ &- \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_3 - r_1)(r_4 - r_1)(r_5 - r_1)(r_6 - r_1)(r_6 - r_0)} + \frac{F(r_2) - F(r_1)}{(r_2 - r_1)(r_3 - r_1)(r_4 - r_1)(r_5 - r_1)(r_6 - r_1)(r_6 - r_0)} \\ &- \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_5 - r_2)(r_5 - r_1)(r_5 - r_0)(r_6 - r_0)} + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_5 - r_2)(r_5 - r_1)(r_5 - r_0)(r_6 - r_0)} \\ &+ \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_5 - r_2)(r_5 - r_1)(r_5 - r_0)(r_6 - r_0)} - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_5 - r_2)(r_5 - r_1)(r_5 - r_0)(r_6 - r_0)} \\ &+ \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_4 - r_1)(r_5 - r_1)(r_5 - r_0)(r_6 - r_0)} - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_4 - r_1)(r_5 - r_1)(r_5 - r_0)(r_6 - r_0)} \\ &- \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_3 - r_1)(r_4 - r_1)(r_4 - r_0)(r_5 - r_0)(r_6 - r_0)} + \frac{F(r_2) - F(r_1)}{(r_2 - r_1)(r_3 - r_1)(r_4 - r_1)(r_4 - r_0)(r_5 - r_0)(r_6 - r_0)} \\ &- \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_3 - r_1)(r_3 - r_0)(r_4 - r_0)(r_5 - r_0)(r_6 - r_0)} + \frac{F(r_2) - F(r_1)}{(r_2 - r_1)(r_3 - r_1)(r_3 - r_0)(r_4 - r_0)(r_5 - r_0)(r_6 - r_0)} \\ &+ \frac{F(r_2) - F(r_1)}{(r_2 - r_1)(r_2 - r_0)(r_3 - r_0)(r_4 - r_0)(r_5 - r_0)(r_6 - r_0)} - \frac{F(r_1) - F(r_0)}{(r_1 - r_0)(r_2 - r_0)(r_3 - r_0)(r_4 - r_0)(r_5 - r_0)(r_6 - r_0)} \end{aligned}$$

$$\begin{aligned}
(13) \quad a_7 = & \frac{F(r_7) - F(r_6)}{(r_7 - r_6)(r_7 - r_5)(r_7 - r_4)(r_7 - r_3)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)} \\
& - \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_7 - r_5)(r_7 - r_4)(r_7 - r_3)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)} \\
& - \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_6 - r_4)(r_7 - r_4)(r_7 - r_3)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)} \\
& + \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_6 - r_4)(r_7 - r_4)(r_7 - r_3)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)} \\
& - \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_6 - r_4)(r_6 - r_3)(r_7 - r_3)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)} \\
& + \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_6 - r_4)(r_6 - r_3)(r_7 - r_3)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)} \\
& + \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_6 - r_3)(r_7 - r_3)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)} \\
& - \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_6 - r_3)(r_7 - r_3)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)} \\
& - \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_6 - r_4)(r_6 - r_3)(r_6 - r_2)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)} \\
& + \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_6 - r_4)(r_6 - r_3)(r_6 - r_2)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)} \\
& + \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_6 - r_3)(r_6 - r_2)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)} \\
& - \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_6 - r_3)(r_6 - r_2)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)} \\
& + \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_5 - r_2)(r_6 - r_2)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)} \\
& - \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_5 - r_2)(r_6 - r_2)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)} \\
& - \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_5 - r_2)(r_6 - r_2)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)} \\
& + \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_5 - r_2)(r_6 - r_2)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)}
\end{aligned}$$

$$\begin{aligned}
(14) \quad & - \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_6 - r_4)(r_6 - r_3)(r_6 - r_2)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)} \\
& + \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_6 - r_4)(r_6 - r_3)(r_6 - r_2)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)} \\
& + \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_6 - r_3)(r_6 - r_2)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)} \\
& - \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_6 - r_3)(r_6 - r_2)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)} \\
& + \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_5 - r_2)(r_6 - r_2)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)} \\
& - \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_5 - r_2)(r_6 - r_2)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)} \\
& - \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_5 - r_2)(r_6 - r_2)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)} \\
& + \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_5 - r_2)(r_6 - r_2)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)} \\
& + \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_5 - r_2)(r_5 - r_1)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)} \\
& - \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_5 - r_2)(r_5 - r_1)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)} \\
& - \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_5 - r_2)(r_5 - r_1)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)} \\
& + \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_5 - r_2)(r_5 - r_1)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)} \\
& - \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_4 - r_1)(r_5 - r_1)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)} \\
& + \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_4 - r_1)(r_5 - r_1)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)} \\
& + \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_3 - r_1)(r_4 - r_1)(r_5 - r_1)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)} \\
& - \frac{F(r_2) - F(r_1)}{(r_2 - r_1)(r_3 - r_1)(r_4 - r_1)(r_5 - r_1)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)}
\end{aligned}$$

$$\begin{aligned}
(15) \quad & - \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_6 - r_4)(r_6 - r_3)(r_6 - r_2)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)} \\
& + \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_6 - r_4)(r_6 - r_3)(r_6 - r_2)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)} \\
& + \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_6 - r_3)(r_6 - r_2)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)} \\
& - \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_6 - r_3)(r_6 - r_2)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)} \\
& + \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_5 - r_2)(r_6 - r_2)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)} \\
& - \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_5 - r_2)(r_6 - r_2)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)} \\
& - \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_5 - r_2)(r_6 - r_2)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)} \\
& + \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_5 - r_2)(r_6 - r_2)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)} \\
& + \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_5 - r_2)(r_5 - r_1)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)} \\
& - \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_5 - r_2)(r_5 - r_1)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)} \\
& - \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_5 - r_2)(r_5 - r_1)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)} \\
& + \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_5 - r_2)(r_5 - r_1)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)} \\
& - \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_4 - r_1)(r_5 - r_1)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)} \\
& + \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_4 - r_1)(r_5 - r_1)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)} \\
& + \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_3 - r_1)(r_4 - r_1)(r_5 - r_1)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)} \\
& - \frac{F(r_2) - F(r_1)}{(r_2 - r_1)(r_3 - r_1)(r_4 - r_1)(r_5 - r_1)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)}
\end{aligned}$$

$$\begin{aligned}
(16) \quad & + \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_5 - r_2)(r_5 - r_1)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)} \\
& - \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_5 - r_2)(r_5 - r_1)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)} \\
& - \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_5 - r_2)(r_5 - r_1)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)} \\
& + \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_5 - r_2)(r_5 - r_1)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)} \\
& - \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_4 - r_1)(r_5 - r_1)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)} \\
& + \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_4 - r_1)(r_5 - r_1)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)} \\
& + \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_3 - r_1)(r_4 - r_1)(r_5 - r_1)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)} \\
& - \frac{F(r_4) - F(r_3)}{(r_2 - r_1)(r_3 - r_1)(r_4 - r_1)(r_5 - r_1)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)} \\
& - \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_4 - r_1)(r_4 - r_0)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)} \\
& + \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_4 - r_1)(r_4 - r_0)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)} \\
& + \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_3 - r_1)(r_4 - r_1)(r_4 - r_0)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)} \\
& - \frac{F(r_2) - F(r_1)}{(r_2 - r_1)(r_3 - r_1)(r_4 - r_1)(r_4 - r_0)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)} \\
& + \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_3 - r_1)(r_3 - r_0)(r_4 - r_0)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)} \\
& - \frac{F(r_2) - F(r_1)}{(r_2 - r_1)(r_3 - r_1)(r_3 - r_0)(r_4 - r_0)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)} \\
& - \frac{F(r_2) - F(r_1)}{(r_2 - r_1)(r_2 - r_0)(r_3 - r_0)(r_4 - r_0)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)} \\
& + \frac{F(r_1) - F(r_0)}{(r_1 - r_0)(r_2 - r_0)(r_3 - r_0)(r_4 - r_0)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)}
\end{aligned}$$

$$\begin{aligned}
(17) \quad a_8 = & \frac{F(r_8) - F(r_7)}{(r_8 - r_7)(r_8 - r_6)(r_8 - r_5)(r_8 - r_4)(r_8 - r_3)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_7) - F(r_6)}{(r_7 - r_6)(r_8 - r_6)(r_8 - r_5)(r_8 - r_4)(r_8 - r_3)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_7) - F(r_6)}{(r_7 - r_6)(r_7 - r_5)(r_8 - r_5)(r_8 - r_4)(r_8 - r_3)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_7 - r_5)(r_8 - r_5)(r_8 - r_4)(r_8 - r_3)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_7) - F(r_6)}{(r_7 - r_6)(r_7 - r_5)(r_7 - r_4)(r_8 - r_4)(r_8 - r_3)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_7 - r_5)(r_7 - r_4)(r_8 - r_4)(r_8 - r_3)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_6 - r_4)(r_7 - r_4)(r_8 - r_4)(r_8 - r_3)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_6 - r_4)(r_7 - r_4)(r_8 - r_4)(r_8 - r_3)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_7) - F(r_6)}{(r_7 - r_6)(r_7 - r_5)(r_7 - r_4)(r_7 - r_3)(r_8 - r_3)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_7 - r_5)(r_7 - r_4)(r_7 - r_3)(r_8 - r_3)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_6 - r_4)(r_7 - r_4)(r_7 - r_3)(r_8 - r_3)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_6 - r_4)(r_7 - r_4)(r_7 - r_3)(r_8 - r_3)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_6 - r_4)(r_6 - r_3)(r_7 - r_3)(r_8 - r_3)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_6 - r_4)(r_6 - r_3)(r_7 - r_3)(r_8 - r_3)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_6 - r_3)(r_7 - r_3)(r_8 - r_3)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_6 - r_3)(r_7 - r_3)(r_8 - r_3)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)}
\end{aligned}$$

$$\begin{aligned}
(18) \quad & - \frac{F(r_7) - F(r_6)}{(r_7 - r_6)(r_7 - r_5)(r_7 - r_4)(r_7 - r_3)(r_7 - r_2)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_7 - r_5)(r_7 - r_4)(r_7 - r_3)(r_7 - r_2)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_6 - r_4)(r_7 - r_4)(r_7 - r_3)(r_7 - r_2)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_6 - r_4)(r_7 - r_4)(r_7 - r_3)(r_7 - r_2)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_6 - r_4)(r_6 - r_3)(r_7 - r_3)(r_7 - r_2)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_6 - r_4)(r_6 - r_3)(r_7 - r_3)(r_7 - r_2)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_6 - r_3)(r_7 - r_3)(r_7 - r_2)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_6 - r_3)(r_7 - r_3)(r_7 - r_2)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_6 - r_4)(r_6 - r_3)(r_6 - r_2)(r_7 - r_2)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_6 - r_4)(r_6 - r_3)(r_6 - r_2)(r_7 - r_2)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_6 - r_3)(r_6 - r_2)(r_7 - r_2)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_6 - r_3)(r_6 - r_2)(r_7 - r_2)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_5 - r_2)(r_6 - r_2)(r_7 - r_2)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_5 - r_2)(r_6 - r_2)(r_7 - r_2)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_5 - r_2)(r_6 - r_2)(r_7 - r_2)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_5 - r_2)(r_6 - r_2)(r_7 - r_2)(r_8 - r_2)(r_8 - r_1)(r_8 - r_0)}
\end{aligned}$$

$$\begin{aligned}
(19) \quad & - \frac{F(r_7) - F(r_6)}{(r_7 - r_6)(r_7 - r_5)(r_7 - r_4)(r_7 - r_3)(r_7 - r_2)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_7 - r_5)(r_7 - r_4)(r_7 - r_3)(r_7 - r_2)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_6 - r_4)(r_7 - r_4)(r_7 - r_3)(r_7 - r_2)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_6 - r_4)(r_7 - r_4)(r_7 - r_3)(r_7 - r_2)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_6 - r_4)(r_6 - r_3)(r_7 - r_3)(r_7 - r_2)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_6 - r_4)(r_6 - r_3)(r_7 - r_3)(r_7 - r_2)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_6 - r_3)(r_7 - r_3)(r_7 - r_2)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_6 - r_3)(r_7 - r_3)(r_7 - r_2)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_6 - r_4)(r_6 - r_3)(r_6 - r_2)(r_7 - r_2)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_6 - r_4)(r_6 - r_3)(r_6 - r_2)(r_7 - r_2)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_6 - r_3)(r_6 - r_2)(r_7 - r_2)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_6 - r_3)(r_6 - r_2)(r_7 - r_2)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_5 - r_2)(r_6 - r_2)(r_7 - r_2)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_5 - r_2)(r_6 - r_2)(r_7 - r_2)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_5 - r_2)(r_6 - r_2)(r_7 - r_2)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_5 - r_2)(r_6 - r_2)(r_7 - r_2)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)}
\end{aligned}$$

$$\begin{aligned}
(20) \quad & + \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_6 - r_4)(r_6 - r_3)(r_6 - r_2)(r_6 - r_1)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_6 - r_4)(r_6 - r_3)(r_6 - r_2)(r_6 - r_1)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_6 - r_3)(r_6 - r_2)(r_6 - r_1)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_6 - r_3)(r_6 - r_2)(r_6 - r_1)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_5 - r_2)(r_6 - r_2)(r_6 - r_1)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_5 - r_2)(r_6 - r_2)(r_6 - r_1)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_5 - r_2)(r_6 - r_2)(r_6 - r_1)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_3 - r_2)(r_4 - r_2)(r_5 - r_2)(r_6 - r_2)(r_6 - r_1)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_4) - F(r_3)}{(r_5 - r_4)(r_5 - r_3)(r_5 - r_2)(r_5 - r_1)(r_6 - r_1)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_5 - r_2)(r_5 - r_1)(r_6 - r_1)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_5 - r_2)(r_5 - r_1)(r_6 - r_1)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_5 - r_2)(r_5 - r_1)(r_6 - r_1)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_4 - r_1)(r_5 - r_1)(r_6 - r_1)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_4 - r_1)(r_5 - r_1)(r_6 - r_1)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_3 - r_1)(r_4 - r_1)(r_5 - r_1)(r_6 - r_1)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)} \\
& + \frac{F(r_2) - F(r_1)}{(r_2 - r_1)(r_3 - r_1)(r_4 - r_1)(r_5 - r_1)(r_6 - r_1)(r_7 - r_1)(r_8 - r_1)(r_8 - r_0)}
\end{aligned}$$

$$\begin{aligned}
& - \frac{F(r_7) - F(r_6)}{(r_7 - r_6)(r_7 - r_5)(r_7 - r_4)(r_7 - r_3)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_7 - r_5)(r_7 - r_4)(r_7 - r_3)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_6 - r_4)(r_7 - r_4)(r_7 - r_3)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_6 - r_4)(r_7 - r_4)(r_7 - r_3)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_6 - r_4)(r_6 - r_3)(r_7 - r_3)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_6 - r_4)(r_6 - r_3)(r_7 - r_3)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_6 - r_3)(r_7 - r_3)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_6 - r_3)(r_7 - r_3)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_6 - r_4)(r_6 - r_3)(r_6 - r_2)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_6 - r_4)(r_6 - r_3)(r_6 - r_2)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_6 - r_3)(r_6 - r_2)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_6 - r_3)(r_6 - r_2)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_5 - r_2)(r_6 - r_2)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_5 - r_2)(r_6 - r_2)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_5 - r_2)(r_6 - r_2)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_5 - r_2)(r_6 - r_2)(r_7 - r_2)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)}
\end{aligned}$$

$$\begin{aligned}
(21) \quad & + \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_6 - r_4)(r_6 - r_3)(r_6 - r_2)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_6 - r_4)(r_6 - r_3)(r_6 - r_2)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_6 - r_3)(r_6 - r_2)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_6 - r_3)(r_6 - r_2)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_5 - r_2)(r_6 - r_2)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_5 - r_2)(r_6 - r_2)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_5 - r_2)(r_6 - r_2)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_5 - r_2)(r_6 - r_2)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_5 - r_2)(r_5 - r_1)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_5 - r_2)(r_5 - r_1)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_5 - r_2)(r_5 - r_1)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_5 - r_2)(r_5 - r_1)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_4 - r_1)(r_5 - r_1)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_4 - r_1)(r_5 - r_1)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_3 - r_1)(r_4 - r_1)(r_5 - r_1)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_2) - F(r_1)}{(r_2 - r_1)(r_3 - r_1)(r_4 - r_1)(r_5 - r_1)(r_6 - r_1)(r_7 - r_1)(r_7 - r_0)(r_8 - r_0)}
\end{aligned}$$

$$\begin{aligned}
(22) \quad & + \frac{F(r_6) - F(r_5)}{(r_6 - r_5)(r_6 - r_4)(r_6 - r_3)(r_6 - r_2)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_6 - r_4)(r_6 - r_3)(r_6 - r_2)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_6 - r_3)(r_6 - r_2)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_6 - r_3)(r_6 - r_2)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_5 - r_2)(r_6 - r_2)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_5 - r_2)(r_6 - r_2)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_5 - r_2)(r_6 - r_2)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_5 - r_2)(r_6 - r_2)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_5 - r_2)(r_5 - r_1)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_5 - r_2)(r_5 - r_1)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_5 - r_2)(r_5 - r_1)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_5 - r_2)(r_5 - r_1)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_4) - F(r_3)}{(r_5 - r_4)(r_5 - r_3)(r_5 - r_2)(r_5 - r_1)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_4 - r_1)(r_5 - r_1)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_4 - r_1)(r_5 - r_1)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_3 - r_1)(r_4 - r_1)(r_5 - r_1)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_2) - F(r_1)}{(r_2 - r_1)(r_3 - r_1)(r_4 - r_1)(r_5 - r_1)(r_6 - r_1)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)}
\end{aligned}$$

$$\begin{aligned}
(23) \quad & - \frac{F(r_5) - F(r_4)}{(r_5 - r_4)(r_5 - r_3)(r_5 - r_2)(r_5 - r_1)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_5 - r_3)(r_5 - r_2)(r_5 - r_1)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_5 - r_2)(r_5 - r_1)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_5 - r_2)(r_5 - r_1)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_4 - r_1)(r_5 - r_1)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_4 - r_1)(r_5 - r_1)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_3 - r_1)(r_4 - r_1)(r_5 - r_1)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_2) - F(r_1)}{(r_2 - r_1)(r_3 - r_1)(r_4 - r_1)(r_5 - r_1)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_4) - F(r_3)}{(r_4 - r_3)(r_4 - r_2)(r_4 - r_1)(r_4 - r_0)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_4 - r_2)(r_4 - r_1)(r_4 - r_0)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_3 - r_1)(r_4 - r_1)(r_4 - r_0)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_2) - F(r_1)}{(r_2 - r_1)(r_3 - r_1)(r_4 - r_1)(r_4 - r_0)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_3) - F(r_2)}{(r_3 - r_2)(r_3 - r_1)(r_3 - r_0)(r_4 - r_0)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_2) - F(r_1)}{(r_2 - r_1)(r_3 - r_1)(r_3 - r_0)(r_4 - r_0)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& + \frac{F(r_2) - F(r_1)}{(r_2 - r_1)(r_2 - r_0)(r_3 - r_0)(r_4 - r_0)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)} \\
& - \frac{F(r_1) - F(r_0)}{(r_1 - r_0)(r_2 - r_0)(r_3 - r_0)(r_4 - r_0)(r_5 - r_0)(r_6 - r_0)(r_7 - r_0)(r_8 - r_0)}
\end{aligned}$$